Performance and Analysis of Switching Function Based Voltage Source Inverter Fed Induction Motor

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Abstract—In industries, the three phase squirrel cage induction motor is widely used due to its simple construction, robust design and low operational costs. The exploitation of squirrel cage induction motor with power semiconductor devices based inverters presents the greater advantages on cost and energy efficiency, compared with other industrial solutions for varying speed applications. Since, Inverter-fed induction motors are gathering great attractiveness for multi-megawatt industrial drive applications these days. In the present paper, an extensive simulation has been carried out for open and closed loop models of 3HP, 50 Hz, 1430 RPM switching function based voltage source inverter fed into induction motor. The transient analysis has been carried out by both models. The analysis has been carried out in the recent MATLAB/Simulink environment. The open and closed loop models give encouraging results and observed that the closed loop model gives better results as compared to open loop model with reduced harmonics. Six motor signatures are used for analysis purpose; these are stator current, rotor current, rotor speed, electromagnetic torque, q-axis rotor flux and q-axis stator flux respectively. Though, it has been seen that the stator current parameter has been widely used parameter in last two decades with Motor Current Signature Analysis (MCSA). Therefore, in the harmonic analysis only stator current parameter is used. The torque control method is used for controlling the induction motor in closed loop due to its advantages over vector or field-control method.

Index Terms—Squirrel Cage Induction Motor (SCIM), Voltage Source Inverter (VSI), Pulse Width Modulation (PWM), Direct Torque Control, Open/Closed Loop, MATLAB/Simulink.

I. INTRODUCTION

The three phase squirrel cage induction motor is simple, efficient and robust asynchronous motor and often a natural choice as a drive for industries with a very competitive pricing. Induction motors are the most extensively used electric motors for appliances, industrial control, automation and transportation. These electric motors frequently called the “workhorse” of the Motion Industry [1-4].

Initially, the induction motors named as constant speed asynchronous motors. But, in the present time a lot of applications require variable speed operations. For example, a washing machine may utilizes different speeds for each wash cycle. Historically, mechanical gear systems were widely employed to achieve variable speed. Recently, electronic power and control systems such as PWM inverter have grown-up to allow these components to be used for motor control in the place of mechanical gears [8-9].

The PWM inverter is not only control the motor’s speed but can also provide improve motor’s dynamic and steady state characteristics. In addition, these devices can reduce the system's average power consumption and noise generation of the induction motor. The exploitation of static frequency inverters are comprehends presently the most efficient method to control the speed of induction motors. Voltage Source Inverters (VSI) Inverters transform a constant frequency constant amplitude voltage into a variable (controllable) frequency-variable (controllable) amplitude voltage. The discrepancy of the power frequency supplied to the motor leads the discrepancy of the rotating of field speed, which modifies the mechanical speed of the induction motor [6,10].

The induction motor is being used these days in several important applications in the place of D.C. motors. The most significant feature which asserts induction motor as a tough competitor to D.C. motor in the drives field is that its cost per KVA is approximately one fifty of its counterpart and it seizes higher appropriateness in antagonistic atmosphere [5,7-8].

Since, the modeling of induction motor has continuously attracted the attention of the researchers because such machines are made and used in leading number of applications. These machines give varied modes of operation in both under dynamic and steady state[12-13].

In electric drive system elements, these machines is a part of the control system elements, which is to be controlled by the dynamic behavior of Induction Motor (IM) then the dynamic model of IM has to be preferred. The dynamic model
considers the instantaneous effects of varying voltage, currents, stator frequency and torque disturbance [8-9].

In the present paper, the open and closed loop switching function based voltage source inverter fed into induction motor model has been developed in the recent MATLAB/Simulink environment. These models are derived by using d and q variables in a stationary rotating reference frame. From these simulation models the transient behavior of the used induction motor has been observed by stator current, rotor current, rotor speed, electromagnetic torque and q-axis stator and rotor fluxes.

II. MATHEMATICAL MODELLING of the MOTOR

The direct-quadrature-zero (dq0) transformation or zero-direct-quadrature (0dq) transformation is a mathematical tool which is used to simplify the analysis of three phase circuits in the electrical engineering. For balanced three phase circuits, the dq0 transform converts three AC quantities into two imaginary DC quantities. Further, simplified calculations can then be carried out on these imaginary DC quantities before performing the inverse transform to recover the actual three-phase AC results. It is frequently used in order to simplify the analysis of three phase induction motors or to simplify calculations for the control of three phase inverters. The mathematical modeling of used induction motor is as shown in fig.1.

The q-axis and d-axis stator voltage equations are given in equation (1) and (2).

\[
V_{qs} = R_s i_{qs} + \frac{d\Psi_{qs}}{dt} + \omega \Psi_{qs},
\]

\[
V_{ds} = R_s i_{ds} + \frac{d\Psi_{ds}}{dt} - \omega \Psi_{qs},
\]

The q-axis and d-axis rotor voltage equations are given in equation (3) and (4).

\[
V'_{qr} = R'_{iqr} + \frac{d\Psi'_qr}{dt} + (\omega - \omega_r)\Psi'_dr,
\]

\[
V'_{dr} = R'_{iqr} + \frac{d\Psi'_dr}{dt} - (\omega - \omega_r)\Psi'_qr,
\]

The flux linkage of q-axis and d-axis for stator and rotor are as given in equations (5 to 8).

\[
\Psi'_{qr} = L_s i_{qr} + L_m i'_{qr},
\]

\[
\Psi'_{dr} = L_s i_{dr} + L_m i'_{dr},
\]

The total stator inductance is given in equation (9)

\[
L_s = L_{s1} + L_{s2},
\]

The total rotor inductance is given in equation (10)

\[
L'_{sr} = L'_{s1} + L_m,
\]

The Electromagnetic Torque is given in equation (11)

\[
T_e = 1.5 p (\Psi'_{dr} i_{qs} - \Psi'_{qr} i_{ds})
\]

The rotor acceleration equation is given in equation (12)

\[
\frac{d\omega_r}{dt} = \frac{1}{2H} (T_e - F \omega_m - T_m)
\]

The angular velocity of the rotor is given in equation (13)

\[
\frac{d\theta_m}{dt} = \omega_m
\]

The quadrature d-axis and q-axis are obtained from abc to dq conversion

A. abc to dq Reference Frame

The abc-to-dq reference frame transformation applied to the induction machine phase-to-phase voltages by equations (14) & (15).

\[
\begin{bmatrix}
V_{qs} \\
V_{ds}
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
2 \cos \theta & \cos \theta + \sqrt{3} \sin \theta \\
2 \sin \theta & \sin \theta - \sqrt{3} \cos \theta
\end{bmatrix}
\begin{bmatrix}
V_{abr} \\
V_{bcr}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V'_{qr} \\
V'_{dr}
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
2 \cos \beta & \cos \beta + \sqrt{3} \sin \beta \\
2 \sin \beta & \sin \beta - \sqrt{3} \cos \beta
\end{bmatrix}
\begin{bmatrix}
V'_{abr} \\
V'_{bcr}
\end{bmatrix}
\]

\[
V_{qs} = R_s i_{qs} + \frac{d\Psi_{qs}}{dt} + \omega \Psi_{qs},
\]

\[
V_{ds} = R_s i_{ds} + \frac{d\Psi_{ds}}{dt} - \omega \Psi_{qs},
\]

\[
V'_{qr} = R'_{iqr} + \frac{d\Psi'_qr}{dt} + (\omega - \omega_r)\Psi'_dr,
\]

\[
V'_{dr} = R'_{iqr} + \frac{d\Psi'_dr}{dt} - (\omega - \omega_r)\Psi'_qr,
\]

\[
\Psi'_{qr} = L_s i_{qr} + L_m i'_{qr},
\]

\[
\Psi'_{dr} = L_s i_{dr} + L_m i'_{dr},
\]

\[
L_s = L_{s1} + L_{s2},
\]

\[
L'_{sr} = L'_{s1} + L_m,
\]

\[
T_e = 1.5 p (\Psi'_{dr} i_{qs} - \Psi'_{qr} i_{ds})
\]

\[
\frac{d\omega_r}{dt} = \frac{1}{2H} (T_e - F \omega_m - T_m)
\]

\[
\frac{d\theta_m}{dt} = \omega_m
\]

\[
\begin{bmatrix}
V_{qs} \\
V_{ds}
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
2 \cos \theta & \cos \theta + \sqrt{3} \sin \theta \\
2 \sin \theta & \sin \theta - \sqrt{3} \cos \theta
\end{bmatrix}
\begin{bmatrix}
V_{abr} \\
V_{bcr}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V'_{qr} \\
V'_{dr}
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
2 \cos \beta & \cos \beta + \sqrt{3} \sin \beta \\
2 \sin \beta & \sin \beta - \sqrt{3} \cos \beta
\end{bmatrix}
\begin{bmatrix}
V'_{abr} \\
V'_{bcr}
\end{bmatrix}
\]
In the previous equations, the angular position of the reference frame is $\theta$ and the difference between the position of the reference frame and the position (electrical) of the rotor is $\beta = \theta - \theta_m$. Since, the machine windings are connected in a three-wire Y configuration and there is no homopolar (0) component. This justifies that the two line-to-neutral input voltages are used instead of three-line-to-neutral voltages.

**B. dq to abc Reference Frame**

Equations (16) and (17) illustrate the $dq$-to-$abc$ reference frame transformations applied to the induction machine phase currents.

\[
\begin{bmatrix}
    i_{ax} \\
    i_{by}
\end{bmatrix}
= \begin{bmatrix}
    \cos \theta & \sin \theta \\
    -\cos \theta + \sqrt{3} \sin \theta & -\sqrt{3} \cos \theta - \sin \theta
\end{bmatrix}
\begin{bmatrix}
    i_{dq} \\
    i_{ds}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    i'_{ar} \\
    i'_{br}
\end{bmatrix}
= \begin{bmatrix}
    \cos \beta & \sin \beta \\
    -\cos \beta + \sqrt{3} \sin \beta & -\sqrt{3} \cos \beta - \sin \beta
\end{bmatrix}
\begin{bmatrix}
    i'_{qr} \\
    i'_{dr}
\end{bmatrix}
\]

\[
i_{cs} = -i_{ax} - i_{by}
\]

\[
i'_{cr} = -i'_{ar} - i'_{br}
\]

Where, $i_{ax}, i_{by}, i_{cs}$ are the stator currents in different phases. $i'_{ar}, i'_{br}, i'_{cr}$ are the rotor current in different phases.

**TABLE I. MACHINE BLOCK PARAMETERS REFERRED TO THE STATOR SIDE**

| $R_s, L_s$ | Stator resistance and leakage inductance | $V_{as}, i_{as}$ | q axis stator voltage and current |
| $R_r, L_r$ | Rotor resistance and leakage inductance | $\theta_m$ | Rotor angular position |
| \(\psi_{q0}, \psi_{d0}\) | Stator q and d axis fluxes | $V_{qs}, i_{qs}$ | q axis rotor voltage and current |
| \(\psi_{qr}, \psi_{dr}\) | Rotor q and d axis fluxes | $\omega_r$ | Electrical angular velocity \((\omega_{m}, x, \beta)\) |
| $L_m$ | Magnetizing inductance | $V_{as}, i_{as}$ | d axis stator voltage and current |
| $p$ | Number of pole pairs | $\theta_r$ | Electrical rotor angular position \((\omega_{m}, x, \beta)\) |
| $L_a, L_d$ | Total stator and rotor inductances | $\omega_m, i_{as}$ | d axis rotor voltage and current |
| $\omega_m$ | Angular velocity of the rotor | $T_e$ | Electromagnetic torque |
| $J_m$ | Shaft mechanical torque | $J$ | Combined rotor, load inertia constant |

**III. PROPOSED OPEN LOOP SWITCHING FUNCTION BASED VSI FED INDUCTION MOTOR**

The induction machine block available in the MATLAB/Simulink operates either in generator mode or motor mode. The modes depend upon the sign of the load torque. If torque is positive motor usually operates in motoring mode if negative generator mode. The mechanical torque is applied on the motor shaft.

The proposed open loop switching function based VSI simulation model of the three phase squirrel cage induction motor is as shown in Fig 2. The applied mechanical rated torque on the shaft is 15 N-m as the full load torque.

The 3 HP, 50Hz, 1430 RPM SCIM is fed into a switching function based VSI PWM inverter for analysis purpose. The fundamental motor frequency is set at 50 Hz. The base frequency of the reference wave is 50 Hz while the triangular carrier wave's frequency is set to 1650 Hz. This corresponds to a frequency modulation factor $m_f$ of 33. The maximum time step has been limited to 10 $\mu$s due to high switching frequency of the inverter. The switching function based VSI inverter is built entirely with standard Simulink blocks and available in MATLAB/Simulink.

Since, the reference frame plays a vital role in the conversion of $abc$ to $dq$ transformations. It is used to convert input voltages ($abc$ reference frame) to the $dq$ reference frame and output currents ($dq$ reference frame) to the $abc$ reference frame in the present paper. The analysis has been carried out...
in the stationary reference frame. Therefore, the value of rotor angle is set to 0 and the value of $\beta$ is set to $-\theta$.

We can choose one among the following reference frame transformations as per our requirement.

i) Rotor reference frame (Park transformation)

ii) Stationary reference frame (Clarke or $\alpha\beta$ transformation)

iii) Synchronous reference frame

The complete mathematical modeling of the used induction motor and corresponding equations of the model have already been explained. The selection of reference frame affects the waveforms of all $dq$ variables. It also affects the simulation speed and in definite cases the accuracy of the results. The following guidelines have been suggested in [5] for choosing the reference frame as follows:

i) Use the stationary reference frame if the stator voltages are either unbalanced or discontinuous and the rotor voltages are balanced (or 0).

ii) Use the rotor reference frame if the rotor voltages are either unbalanced or discontinuous and the stator voltages are balanced.

iii) Use either the stationary or synchronous reference frames if all voltages are balanced and continuous.

The brief description about the induction motor parameters is as shown in Table: II.

<table>
<thead>
<tr>
<th>Rotor Type: Squirrel Cage</th>
<th>Reference Frame: Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor: 3 HP</td>
<td>DC Voltage: 400V</td>
</tr>
<tr>
<td>Frequency: 50 Hz</td>
<td>Stator Resistance: 0.435 Ω</td>
</tr>
<tr>
<td>Rotor Resistance: 0.816 Ω</td>
<td>Mutual Inductance: 69.31 ×10⁻³ H</td>
</tr>
<tr>
<td>No. of Pole: 4</td>
<td>Speed: 1430 RPM</td>
</tr>
</tbody>
</table>

A. Simulation Results of Open Loop Model

The simulation results in symmetrical (or called healthy mode) motor condition of the motor are as shown in Fig. 3. Six motor signatures have been considered for analysis purpose, these are stator current, rotor current, rotor speed, electromagnetic torque, q-axis stator/rotor flux. The slip and load torque are set 1 and 15 N-m respectively in the healthy condition.

In the healthy condition of the motor the slip is set at 1(s=1) and input mechanical torque is 15 N-m. The obtained results clearly show, all the motor parameters have been reached in the steady state condition after 0.4 seconds. It has been observed that in the starting of the motor, the first peak of the stator current and electromagnetic torque waveforms has highest amplitudes when it is decreased and reached in the stable condition.

The model is simulated for 1 sec for clear visualization of transient characteristics. Therefore, the transient characteristics of the motor may be clearly observed and analysed.

The motor starts and reaches its steady state speed 1430 rpm, after 0.4 sec. At starting, the magnitude of the 50 Hz current reaches 96 A peak (68 A RMS) whereas its steady state value is 12.25 A (8.66 A RMS).

The electromagnetic torque waveform shows that it is reached in the stable condition after 0.4 sec. It has also been observed that the strong oscillations of the electromagnetic torque at starting. If it is zoom in on the torque in steady state, it will be observed a noisy signal with a mean value of 15 N.m, corresponding to the load torque at nominal speed.
IV. PROPOSED CLOSED LOOP SWITCHING FUNCTION BASED VSI FED INDUCTION MOTOR

The closed loop switching function based VSI fed induction motor simulation model is as shown in Fig. 4. The torque control method has been used with inverter for controlling the induction motor. The field oriented control or vector control method can also be used with PWM inverter and can obtain same flexibility in the speed and torque control like DC machines. The field control technique is an attractive control technique but it faces serious problem that it relies heavily on precise acquaintance of the motor parameters. The rotor time constant is particularly complicated to measure precisely, and to make stuff inferior because it varies with temperature. Since, the torque control method is efficiently controls the induction motor. It estimates the stator flux and electric torque in the stationary.

If we zoom on the three motor currents, we may observe that all the harmonics (multiple of the 1650 Hz switching frequency) are filtered by the stator inductance. Hence we can say that the 50 Hz component is dominant.

If we observed $q$-axis stator and rotor flux, it gives perfectly sinusoidal flux after 0.4 sec. Therefore, it can be concluded that the proposed model give excellent results as expected.

Fig. 4 Simulation Model of Closed Loop Switching Function Based VSI Fed Induction Motor

reference frame and terminal measurements. The equations associated with closed loop simulation model are as follows:

\[
\Psi_{ds} = \int (V_{ds} - R_i i_{ds}) dt
\]  
(20)

\[
\Psi_{qs} = \int (V_{qs} - R_i i_{qs}) dt
\]  
(21)

\[
\phi_s = \sqrt{\Psi_{ds}^2 + \Psi_{qs}^2} \leq \tan^{-1}\left(\frac{\Psi_{qs}}{\Psi_{ds}}\right)
\]  
(22)

\[
T_c = 1.5 p (\Psi_{ds} i_{qs} - \Psi_{qs} i_{ds})
\]  
(23)

The estimated stator flux and electric torque are controlled directly by comparing them with their respective demanded values using the hysteresis comparators. The outputs of the two comparators are used as input signals of an optional switching table. The rotor speed is fed back and directly

\[
R_i = \frac{1}{L_s} \frac{d\Psi_s}{dt}
\]  
(19)
connected to the shaft. The value of the feedback constant \( k \) is \( 6.693 \times 10^{-4} \).

**A. Simulation Results of Closed Loop Model**

Fig. 5 (a) Stator Current, (b) Rotor Current, (c) Rotor Speed, (d) Electromagnetic Torque, (e) q-axis Stator Flux, (f) q-axis Rotor Flux

**V. HARMONIC ANALYSIS of OPEN and CLOSED LOOP MODELS**

The Total Harmonic Distortion (THD) analysis for open and closed loop simulation models has been discussed for its stator current and input line-to-line voltage. The waveforms of stator current for 1000Hz maximum frequency is as shown in Fig. 6 and Fig. 7 for open loop and closed loop respectively. In this THD analysis, we have sampling frequency 2 KHz.
Therefore, a DC voltage of 400 V and a modulation factor of 0.90 yields the 220 V_{dc} output line-to-line voltage, which is the required voltage of the induction motor.

$$V_{LL_{rms}} = \frac{m}{2} \times \sqrt{\frac{3}{2}} V_{dc} = m \times 0.612 \times V_{dc}$$  \hspace{1cm} (24)

The THD in open loop stator current is more in comparison to closed loop but for the line-to-line voltage, THD is same in both the cases (open/close). It is because we are not controlling input line-to-line voltage. Therefore, PWM inverter output voltage will be unchanged in open loop as well as closed loop. It has been observed that if the inverter is fed into induction motor then line voltage will be disturbed consequently rise in THD for input-line-to-line voltage but THD in the stator current will be less.

The fundamental component and Total Harmonic Distortion (THD) of the $V_{ab}$ voltage are displayed above the spectrum window. The magnitude of the fundamental of the inverter voltage (312.1 V) is compared well with the theoretical value of $m=0.9(312.1V)$ as shown in Fig.6(b) and Fig.7(b). The line-to-line RMS voltage is a function of the DC input voltage and of the modulation index $m$ as given by the following equation:

V. CONCLUSIONS

In the present paper, the extensive simulation has been carried out in the recent MATLAB/Simulink environment for 3 HP, 4 pole, 50 Hz open and closed loop induction motor connected with switching function based inverter. It has been observed that the closed loop model has given better results as compared to open loop model with reduced harmonics. The torque control method has been used for induction motor controlling purpose over field-oriented control technique due to its advantages. From both models, the transient behavior of the used induction motor has been observed. It has also been observed that the switching function based inverter influences the motor performance and might introduce disturbances into
the main power line. But precisely designed interface system would be very useful in drive control applications. These models may be used in various power electronics and drives applications in industry.

REFERENCES


ACKNOWLEDGMENT

Authors acknowledge Technical Education Quality Improvement Program Phase-II, Institute of Engineering & Technology, Lucknow for providing financial assistantship to carry out the research work.